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Aberration of Light from Binary Stars—a Paradox?

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It is widely believed that aberration, like the Doppler effect, depends on the relative velocity of source and observer. It is here shown that, if this were true, binary stars would mostly look widely separated and rapidly rotating. Not only is this not observed, but it would appear to conflict with Kepler's third law if it were. It is argued that aberration does not depend on the relative velocities of source and observer: it depends only on the change in velocity of the observer between the times when the two measurements from which the aberration is deduced are made. The misconception is due to a faulty customary interpretation of the correct standard treatment.

SUPPOSE a distant observer 0 looks at two stars S_1, S_2 rotating about the center of mass C of the pair. For simplicity we shall suppose C is at rest relative to 0, that the orbits of the stars are circular, and that 0 lies on the normal through C to the plane of the orbits. None of these simplifications makes any essential difference to the arguments.

Let the distance of the observer from the center of mass be D, the velocities of the two stars at the moment of observation $\mathbf{v}_1$, $\mathbf{v}_2$, the radii of their orbits R_1, R_2, and let c be the speed of light. Let us assume that we may consider the stars to be instantaneously in uniform motion with respect to the observer, and that gravitational fields are small, so that use of the special theory of relativity is legitimate. Then if the aberration depends only on the relative velocities of source and observer, the apparent directions of the stars as seen from 0 must be displaced from their true directions by vectors $-\mathbf{v}_1/c$, $-\mathbf{v}_2/c$ (to first order in v/c) (see Fig. 1). It can readily be seen that the stars must appear to describe circular orbits of angular radii ρ_1, ρ_2 where

$$\rho_1=\left[\left(\frac{v_1}{c}\right)^2+\left(\frac{R_1}{D}\right)^2\right]^{1/2}=\frac{R_1}{cT}\left[1+\left(\frac{c'T}{D}\right)^2\right]^{1/2}=\frac{Km_2}{T^{1/3}}\left[1+\left(\frac{c'T}{D}\right)^2\right]^{1/2}$$

$$\rho_2=\left[\left(\frac{v_2}{c}\right)^2+\left(\frac{R_2}{D}\right)^2\right]^{1/2}=\frac{R_2}{cT}\left[1+\left(\frac{c'T}{D}\right)^2\right]^{1/2}=\frac{Km_1}{T^{1/3}}\left[1+\left(\frac{c'T}{D}\right)^2\right]^{1/2}$$

($v_1=|\mathbf{v}_1|$, $v_2=|\mathbf{v}_2|$; T is the period of revolution; $c'=c/2\pi$; m_1, m_2 are the masses of the stars; $K=[2\pi G/(m_1+m_2)^2]^{1/3}/c$, where G is the gravitational constant.)

This result is paradoxical. For, if the system were observed by very distant observers, the orbits would have *angular* radii v_1/c, v_2/c, in-

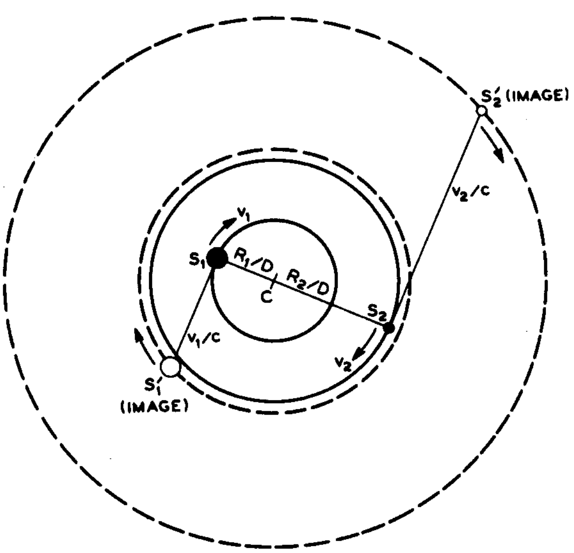

FIG. 1. Angular relations of the true and apparent positions and orbits of binary stars, if aberration depends on relative velocity of source and observer. ——: True orbits; – – –: apparent orbits.

dependent of distance. Furthermore, v/c is commonly of order 10^{-4} ($\equiv 20''$) and about one "star" in three is a multiple system; so that the skies should be filled with binary stars of apparent separation of order $40''$. Indeed, such pairs should be seen circling distant galaxies many of which appear much smaller than that! And the apparent angular diameters of the orbits would increase with decreasing period, which would be very spectacular and, incidentally, contrary to Kepler's third law.

In fact there are very few pairs known of separation greater than $10''$, and these move very slowly.

The paradox follows direct from the assumption that the aberration depends only on the relative motion of source and observer. That this is so is stated explicitly or implicitly in most treatments of aberration,[1–4] and indeed is sometimes emphasized[2] as distinguishing aberration in special relativity from aberration in "æther" theories. In a stationary æther, the paradox would not arise.

Consider two sources of light moving relative to each other, and suppose they coincide at a certain moment. Suppose they each emit a pulse of light at that moment. The two pulses of light are then coincident in space and time for observers at rest with respect to either source and will appear coincident to all observers moving uniformly with respect to them. Let us put it another way: consider the light rays from each of the sources that will eventually reach a given observer. These two rays pass through a given point in space in the observer's system at the sources, and through another given point at the observer. If the rays are straight, then they must coincide.

Thus, the observer sees the two sources in the same direction, and so this direction does not depend on the velocity of the source relative to the observer. He sees a source of light in the direction of the point in space in his coordinate system that the source occupied when it emitted the light. Of course, the source may no longer be at that point when the light reaches the observer: it may even have disintegrated in the interval.

I contend, then, that both "secular aberration" and "planetary aberration,"[5] denote effects physically different from that denoted by "annual aberration" and "diurnal aberration."

To see what we do mean by "aberration," we must analyze the experiment in which it is observed. We observe the change in apparent direction of a star with a change in the *observer's* velocity. If the star is far enough away, this is equivalent to comparing the apparent directions in two systems, 1 and 2, coincident in space and time, when 2 moves with speed $c\beta$ relative to 1. The usual treatment[1–4] then shows that

$$\tan \alpha_2 = (1 - \beta^2)^{1/2} \sin \alpha_1 / (\cos \alpha_1 + \beta)$$

[For small β, $\alpha_2 - \alpha_1 = -\beta \sin \alpha_1 + \frac{1}{4} \beta^2 \sin 2\alpha_1 + \cdots$], where the alphas are the angles between the observed directions and the velocity of 2 with respect to 1. The difference $(\alpha_2 - \alpha_1)$ is properly called "aberration."

If Systems 1 and 2 are at rest relative to the centers of the earth and the sun, respectively, then $c\beta$ is the orbital velocity of the earth and $(\alpha_2 - \alpha_1)$ is the "annual aberration." If Systems 1 and 2 were at rest relative to the sun and to a star, respectively, then one could reasonably call $(\alpha_2 - \alpha_1)$ the "secular aberration" of the star; but α_2 would have no simple, geometrical meaning. Instead, "secular aberration" is defined as the angle $(\alpha_3 - \alpha_1)$ between the apparent

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1 A. Einstein, "Zur Elektrodynamik bewegter Körper," Ann. Physik **17**, 891–921 (1905) (English translation by W. Perrett and G. B. Jeffrey, Methuen, London, 1923). The angle φ' used on p. 912 is stated to be the angle between the ray direction and the line joining the source and the observer, in the observer's system: the results are correct only if φ' is the angle between the ray direction in the observer's system and the direction of the relative velocity of the two systems. Even then, as discussed in the present paper, $(\varphi - \varphi')$ is not necessarily the aberration.

2 W. Pauli, *Theory of Relativity* (1921; English translation by G. Field, Pergamon Press, Ltd., London, 1958), Sec. 6.

3 G. Joos, *Theoretical Physics* (English translation (first edition) by I. M. Freeman (Blackie & Sons, London, 1934), p. 233.

4 C. Møller, *The Theory of Relativity* (Oxford University Press, London, 1955).

5 *The Explanatory Supplement to the Astronomical Ephemeris and the American Ephemeris and Nautical Almanac* (Her Majesty's Stationary Office, London, 1961) Chap. 2D.

direction of a star and its true direction at the time of observation, for an observer at rest relative to the sun. The true direction cannot be measured until the light then emitted reaches the sun. Further, $(\alpha_3 - \alpha_1)$ has a different physical origin from $(\alpha_2 - \alpha_1)$. We see this at once if, as with the binary stars, the star does not move uniformly with respect to the sun; but also with uniform motion, they are not even formally the same:

$$\tan \alpha_3 = \sin \alpha_1/(\cos \alpha_1 + \beta)$$

[For small β, $(\alpha_3 - \alpha_1) = -\beta \sin \alpha_1 + \frac{1}{2}\beta^2 \sin 2\alpha_1 + \cdots$.] The fact that we believe that the source is no longer where it was when it emitted the light we observe says nothing about transformation between frames of reference, nor about the properties of light, except that it has a finite speed. It says instead that we have a dynamical model of the motion of the source with respect to the observer. This has nothing to do with "aberration."

In our example with binary stars, the observer simply sees the stars where they were when the light left—as one would expect intuitively. The concept implied by the term *secular aberration* leads to the paradox.

Aberration is unsymmetrical with respect to the velocities of observer and source, because the experiment that measures it requires an unsymmetrical situation. While we cannot determine whether the source or the observer is in motion, there is no doubt which has *changed* its state of uniform motion.

[A warning is perhaps called for to anyone who might try relativistically compounding the velocities of star and earth relative to the sun and then transforming the velocity of a photon from the star's system into the earth's. The Lorentz transformation without rotation of the coordinate axes is not a subgroup of the Lorentz group (Ref. 4, Sec. 22). So, if we define a set of coordinates in the earth's system parallel to that in the sun's and a set in the star's system also parallel to that in the sun's, then the star's coordinates will *not,* in general, appear parallel to the earth's. In particular, the earth's velocity in the star's system will *not* be the negative of the star's velocity in the earth's system; and it will not be legitimate to use a Lorentz transformation without rotation, for transformation from the star's system direct into the earth's.]

We have shown that the "binary star paradox" is not a paradox at all, but merely a consequence of an erroneous, but commonly accepted, interpretation of the Lorentz transformation as applied to aberration. The fact that we do not see myriads of widely separated binaries in wild gyration does not require any fundamental change of outlook. But it does require that aberration should be realistically treated as the transformation between the frames of references of two observers, not a source and an observer.[6] This would show that the difference between the "true" direction of a star and its observed direction is not aberration at all. The replacement of "secular aberration" by some term such as "light-time lag" would help this realisation.

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[6] J. L. Synge, *Relativity: the Special Theory* (North-Holland Publishing Co., Amsterdam, 1965), 2nd Edition, p. 145.